# Engineering Notes

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## Nonlinear Dynamic Equations of Satellite Relative Motion Around an Oblate Earth

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### I. Introduction

A $J_2$ dynamic model of satellite relative motion in the form of differential equations with reference to the local vertical local horizontal (LVLH) coordinate is basic and critical to the study of satellite formation flying. Many versions [1–7] of $J_2$ dynamic equations have been published. The work [1] by Kechichian presents an exact nonlinear relative model taking into account both $J_2$ perturbation and air drag. Kechichian applied techniques of Newtonian mechanics and vector calculus to derive the relative dynamics. However, his model is very complex and does not explicitly include some critical components of the $J_2$ acceleration. The calculation of these $J_2$ acceleration components is by a tedious algorithm. The model's complexity hampers its application in control designs. Other published dynamic models [2–7] were developed assuming the reference orbit as being unperturbed Keplerian motion; thus, modeling errors are introduced.

In this Note, the satellite relative dynamics is studied based on the perturbed reference orbit, which is accurately described by a set of differential equations. We express the reference orbit in terms of reference satellite variables (RSV) and obtain a simpler representation than that by Kechichian. Furthermore, we use Lagrangian mechanics to derive the satellite relative dynamics, which is different from Kechichian's approach. As a result, we establish an exact $J_2$ nonlinear relative model independent of the right ascension of the ascending node. The satellite relative motion is explicitly expressed in terms of very few physical parameters and a set of simple first-order differential equations. This simplicity is due to the fact that both the spherical and the $J_2$ accelerations are independent of the change of the right ascension of the ascending node.

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### II. $J_2$ Reference Satellite Dynamics in a Rotating Frame

In this study, one satellite, or a virtual satellite, is taken as the reference satellite and the others as member satellites. Without loss of generality, we consider a two-satellite system, that is, a free-flying reference satellite $S_0$ (without control force) and a controlled member satellite $S_j$ (with control force). This section is devoted to establishing the $J_2$ dynamics of the reference satellite $S_0$ in the local rotating frame.

Two Cartesian coordinate frames are used as shown in Fig. 1. The Earth centered inertial (ECI) coordinate frame is spanned by the unit vectors $(\hat{\mathbf{X}}, \hat{\mathbf{Y}}, \hat{\mathbf{Z}})$. The LVLH coordinate frame is attached on the reference satellite $S_0$. We denote the position and the velocity of the satellite $S_0$ by the vectors $\mathbf{r}$ and $\dot{\mathbf{r}}$, respectively. The vector of the angular momentum per unit mass is defined as $\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}}$. Furthermore, we denote the geocentric distance and the angular momentum magnitude of the reference satellite $S_0$ as $r = |\mathbf{r}|$ and $h = |\mathbf{h}|$. Then, the LVLH coordinate is spanned by the unit vectors

$$\hat{\mathbf{x}} = \mathbf{r}/r \qquad \hat{\mathbf{y}} = \hat{\mathbf{z}} \times \hat{\mathbf{x}} \qquad \hat{\mathbf{z}} = \mathbf{h}/h \tag{1}$$

The angular velocity of the rotating LVLH frame is

$$\boldsymbol{\omega} = \omega_x \hat{\mathbf{x}} + \omega_z \hat{\mathbf{z}} \tag{2}$$

whose component around the $y$ axis is zero [1], that is, $\omega_y = 0$. We refer to $\omega_x$ as the steering rate of the orbital plane and refer to $\omega_z$ as the orbital rate, which can be computed [1,8] by

$$\omega_z = h/r^2 \tag{3}$$

The angular velocity $\boldsymbol{\omega}$ can also be expressed by the Eulerian angles $(\Omega, i, \theta)$, and its three components are [1,8]

$$\omega_x = \dot{i} c_\theta + \dot{\Omega} s_\theta s_i \tag{4}$$

$$\omega_y = -\dot{i} s_\theta + \dot{\Omega} c_\theta s_i = 0 \tag{5}$$

$$\omega_z = \dot{\theta} + \dot{\Omega} c_i \tag{6}$$

In this Note, we use the notations $s_\circ = \sin(\circ)$ and $c_\circ = \cos(\circ)$. Using Eq. (2), the velocities of the unit vectors of the LVLH frame are expressed as

$$\dot{\hat{\mathbf{x}}} = \boldsymbol{\omega} \times \hat{\mathbf{x}} = \omega_z \hat{\mathbf{y}} \qquad \dot{\hat{\mathbf{y}}} = \boldsymbol{\omega} \times \hat{\mathbf{y}} = \omega_x \hat{\mathbf{z}} - \omega_z \hat{\mathbf{x}} \qquad \dot{\hat{\mathbf{z}}} = \boldsymbol{\omega} \times \hat{\mathbf{z}} = -\omega_x \hat{\mathbf{y}} \tag{7}$$

With these properties of the LVLH frame, the dynamics of the reference satellite $S_0$ under $J_2$ perturbation can be derived. It is presented in the following Proposition. For clear presentation, we define the following constant:

$$k_{J2} = 3 J_2 \mu R_e^2 / 2 \tag{8}$$

where $\mu$ is the Earth's gravitational constant, $J_2$ is the second zonal harmonic coefficient of the Earth, and $R_e$ is the Earth's equatorial radius.

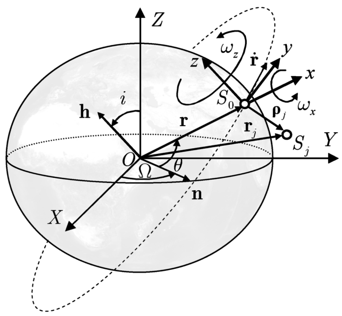


**Fig. 1 ECI and LVLH coordinate frames.**

*Proposition 1*: Considering the spherical gravity and the $J_2$ gravity of the Earth, the motion of the reference satellite $S_0$ can be described by a set of differential equations as follows:

$$\dot{r} = v_x \tag{9}$$

$$\dot{v}_x = -\frac{\mu}{r^2} + \frac{h^2}{r^3} - \frac{k_{J2}}{r^4}(1 - 3s_i^2 s_\theta^2) \tag{10}$$

$$\dot{h} = -\frac{k_{J2} s_i^2 s_{2\theta}}{r^3} \tag{11}$$

$$\dot{\theta} = \frac{h}{r^2} + \frac{2k_{J2} c_i^2 s_\theta^2}{hr^3} \tag{12}$$

$$\dot{i} = -\frac{k_{J2} s_{2i} s_{2\theta}}{2hr^3} \tag{13}$$

$$\dot{\Omega} = -\frac{2k_{J2} c_i s_\theta^2}{hr^3} \tag{14}$$

*Proof*: Considering the spherical gravity and the $J_2$ gravity, the motion of the satellite $S_0$ is governed by the following equation:

$$\ddot{\mathbf{r}} = -\nabla U \tag{15}$$

where the gravitational potential $U$ is

$$U = -\frac{\mu}{r} - \frac{k_{J2}}{r^3}\left(\frac{1}{3} - s_\phi^2\right) \tag{16}$$

with $\phi$ being the geocentric latitude. Considering Eqs. (3) and (7), the acceleration vector $\ddot{\mathbf{r}}$ of the satellite $S_0$ is derived by taking the time derivatives twice to the position vector $\mathbf{r} = r\hat{\mathbf{x}}$ and expressed as

$$\ddot{\mathbf{r}} = \left(\dot{v}_x - \frac{h^2}{r^3}\right)\hat{\mathbf{x}} + \frac{\dot{h}}{r}\hat{\mathbf{y}} + \frac{\omega_x h}{r}\hat{\mathbf{z}} \tag{17}$$

where the radial velocity $v_x = \dot{r}$ is defined and hence Eq. (9) is established. On the other hand, using the constant defined in Eq. (8) and the fact $s_\phi = Z/r$, the gradient of $U$ is obtained as follows:

$$\nabla U = \left(\frac{\mu}{r^3} + \frac{k_{J2}}{r^5}(1 - 5s_\phi^2)\right)\mathbf{r} + \frac{2k_{J2} s_\phi}{r^4}\hat{\mathbf{Z}} \tag{18}$$

Furthermore, using the fact ™

$$\hat{\mathbf{Z}} = s_\theta s_i \hat{\mathbf{x}} + c_\theta s_i \hat{\mathbf{y}} + c_i \hat{\mathbf{z}} \tag{19}$$

as well as $s_\phi = s_\theta s_i$ and $\mathbf{r} = r\hat{\mathbf{x}}$, Eq. (18) becomes

$$\nabla U = \frac{\mu}{r^2}\hat{\mathbf{x}} + \frac{k_{J2}}{r^4}(1 - 3s_i^2 s_\theta^2)\hat{\mathbf{x}} + \frac{k_{J2} s_i^2 s_{2\theta}}{r^4}\hat{\mathbf{y}} + \frac{k_{J2} s_{2i} s_\theta}{r^4}\hat{\mathbf{z}} \tag{20}$$

Substituting Eqs. (17) and (20) into Eq. (15), we establish Eqs. (10) and (11) and obtain

$$\omega_x = -\frac{k_{J2} s_{2i} s_\theta}{hr^3} \tag{21}$$

Replacing $\omega_x$ in Eq. (4) with Eq. (21), the dynamics(13) and (14) of $(i, \Omega)$ are established from Eqs. (4) and (5). Finally, using Eqs. (3) and (14) into Eq. (6), the dynamics (12) of $\theta$ is established. Clearly, six variables $(r, v_x, h, \theta, i, \Omega)$ are solutions of the dynamics (9–14) and completely describe the motion of the satellite $S_0$. □

To the dynamics in Proposition 1, we have the following Remark.

*Remark 1*: We refer to the six variables $(r, v_x, h, \theta, i, \Omega)$ as reference satellite variables. We notice that the dynamics (9–13) are independent of the right ascension of the ascending node $\Omega$. This is because both the spherical and the $J_2$ gravitational potentials are symmetric to the Earth's rotation axis $\hat{\mathbf{Z}}$. Thus, we refer to the five variables $(v_x, h, r, \theta, i)$ as compact reference satellite variables (CRSV). □

The rotation of the LVLH frame can be conveniently expressed in terms of CRSV. Taking time derivatives to the orbital rate $\omega_z$ in Eq. (3) and the steering rate $\omega_x$ in Eq. (21), respectively, and using Eqs. (9) and (11–13), the orbital acceleration $\alpha_z$ and the steering acceleration $\alpha_x$ are obtained as

$$\alpha_z = \dot{\omega}_z = -\frac{2hv_x}{r^3} - \frac{k_{J2} s_i^2 s_{2\theta}}{r^5} \tag{22}$$

$$\alpha_x = \dot{\omega}_x = -\frac{k_{J2} s_{2i} c_\theta}{r^5} + \frac{3v_x k_{J2} s_{2i} s_\theta}{r^4 h} - \frac{8k_{J2}^2 s_i^3 c_i s_\theta^2 c_\theta}{r^6 h^2} \tag{23}$$

## III. Derivation of Exact $J_2$ Nonlinear Relative Dynamics

### A. Lagrangian Formulation of Relative Motion

In this section, we use the Lagrangian formulation to develop the relative dynamics of the member satellite $S_j$. The Lagrangian formulation for the satellite relative motion is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L_j}{\partial \dot{\mathbf{q}}_j}\right) - \frac{\partial L_j}{\partial \mathbf{q}_j} = \mathbf{F}_j \tag{24}$$

where $\mathbf{q}_j = [x_j \quad y_j \quad z_j]^T$ and $\mathbf{F}_j = [F_{jx} \quad F_{jy} \quad F_{jz}]^T$ are, respectively, the configurations and the control forces of the satellite $S_j$ in the LVLH coordinate and $L_j$ is its Lagrangian, which can be expressed in the form of

$$L_j(\mathbf{q}_0, \dot{\mathbf{q}}_0, \mathbf{q}_j, \dot{\mathbf{q}}_j) = K_j(\mathbf{q}_0, \dot{\mathbf{q}}_0, \mathbf{q}_j, \dot{\mathbf{q}}_j) - U_j(\mathbf{q}_0, \mathbf{q}_j) \tag{25}$$

where $\mathbf{q}_0$ is the configuration vector of the reference satellite $S_0$ in the ECI frame. $K_j$ and $U_j$ are, respectively, the kinetic and potential energies of the $j$th member satellite. Because the kinetic energy $K_j$ should be expressed in terms of inertial motion, it depends on not only the relative motion $(\mathbf{q}_j, \dot{\mathbf{q}}_j)$ of the satellite $S_j$ in the LVLH frame but also the transport motion $(\mathbf{q}_0, \dot{\mathbf{q}}_0)$ of the LVLH frame in the ECI frame. On the other hand, the potential energy $U_j$ is solely due to gravity and thus is independent of velocities. Substituting Eq. (25) into Eq. (24) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial K_j}{\partial \dot{\mathbf{q}}_j}\right) - \frac{\partial K_j}{\partial \mathbf{q}_j} + \frac{\partial U_j}{\partial \mathbf{q}_j} = \mathbf{F}_j \tag{26}$$

which is the Lagrangian formulation for the relative motion of the member satellite $S_j$ in the LVLH frame. The keys to use the formulation Eq. (26) are the precise calculations of the kinetic energy $K_j$ and the potential energy $U_j$.

### B. Kinetic Energy

The position vector of the satellite $S_j$, as shown in Fig. 1, is

$$\mathbf{r}_j = \mathbf{r} + \boldsymbol{\rho}_j = r\hat{\mathbf{x}} + (x_j\hat{\mathbf{x}} + y_j\hat{\mathbf{y}} + z_j\hat{\mathbf{z}}) \tag{27}$$

Using identities in Eq. (7), the velocity vector $\dot{\mathbf{r}}_j$ can be calculated by taking the time derivative of (27). Then, the kinetic energy per unit mass of the satellite $S_j$ is

$$K_j = \frac{1}{2}\dot{\mathbf{r}}_j \cdot \dot{\mathbf{r}}_j = \frac{1}{2}(\dot{x}_j + v_x - y_j\omega_z)^2 + \frac{1}{2}(\dot{y}_j + (r + x_j)\omega_z - z_j\omega_x)^2 + \frac{1}{2}(\dot{z}_j + y_j\omega_x)^2 \tag{28}$$

where the variables $(r, v_x, \omega_x, \omega_z)$ are functions of CRSV.

### C. Potential Energy

Considering the $J_2$ perturbation, the gravitational potential energy of the satellite $S_j$ is

$$U_j = -\frac{\mu}{r_j} - \frac{k_{J2}}{r_j^3}\left(\frac{1}{3} - s_{\phi_j}^2\right) \tag{29}$$

where $\phi_j$ and $r_j$ are the geocentric latitude and the geocentric distance of the satellite $S_j$, respectively. From Eq. (27), the geocentric distance $r_j$ is obtained as

$$r_j = |\mathbf{r}_j| = \sqrt{(r + x_j)^2 + y_j^2 + z_j^2} \tag{30}$$

The physical meaning of the geocentric latitude $\phi_j$ leads to

$$s_{\phi_j} = r_{jZ}/r_j \tag{31}$$

where $r_{jZ}$ is the projection of $\mathbf{r}_j$ on the $Z$ axis of the ECI frame. Considering Eqs. (19) and (27), $r_{jZ}$ is computed to

$$r_{jZ} = \mathbf{r}_j \cdot \hat{\mathbf{Z}} = (r + x_j)s_i s_\theta + y_j s_i c_\theta + z_j c_i \tag{32}$$

Substituting Eq. (31) into Eq. (29), the potential energy of the member satellite $S_j$ is obtained as

$$U_j = -\frac{\mu}{r_j} - \frac{k_{J2}}{3r_j^3} + \frac{k_{J2}r_{jZ}^2}{r_j^5} \tag{33}$$

where $r_j$ and $r_{jZ}$ are given in Eqs. (30) and (32).

### D. Exact Nonlinear $J_2$ Relative Dynamics

Using the kinetic energy [Eq. (28)] and the potential energy [Eq, (33)] into the Lagrangian formulation [Eq. (26)], the nonlinear dynamic equations of the satellite relative motion can be derived and presented in the following Proposition.

*Proposition 2*: Consider a two-satellite system of the reference satellite $S_0$ and the member satellite $S_j$, as shown in Fig. 1. In the presence of the spherical gravity and the $J_2$ gravity of the Earth, the relative motion of the satellite $S_j$ in the LVLH frame can be described by

$$\begin{aligned} \ddot{x}_j &= 2\dot{y}_j\omega_z - x_j(\eta_j^2 - \omega_z^2) + y_j\alpha_z - z_j\omega_x\omega_z - (\zeta_j - \zeta)s_i s_\theta - r(\eta_j^2 - \eta^2) + F_{jx}, \\ \ddot{y}_j &= -2\dot{x}_j\omega_z + 2\dot{z}_j\omega_x - x_j\alpha_z - y_j(\eta_j^2 - \omega_z^2 - \omega_x^2) + z_j\alpha_x - (\zeta_j - \zeta)s_i c_\theta + F_{jy}, \\ \ddot{z}_j &= -2\dot{y}_j\omega_x - x_j\omega_x\omega_z - y_j\alpha_x - z_j(\eta_j^2 - \omega_x^2) - (\zeta_j - \zeta)c_i + F_{jz} \end{aligned} \tag{34}$$

where $(F_{jx}, F_{jy}, F_{jz},)$ are control forces on the satellite $S_j$. The accelerations $(\zeta, \zeta_j)$ are

$$\zeta = \frac{2k_{J2}s_i s_\theta}{r^4} \tag{35}$$

$$\zeta_j = \frac{2k_{J2}r_{jZ}}{r_j^5} \tag{36}$$

The angular velocities $(\eta, \eta_j)$ can be obtained from

$$\eta^2 = \frac{\mu}{r^3} + \frac{k_{J2}}{r^5} - \frac{5k_{J2}s_i^2 s_\theta^2}{r^5} \tag{37}$$

$$\eta_j^2 = \frac{\mu}{r_j^3} + \frac{k_{J2}}{r_j^5} - \frac{5k_{J2}r_{jZ}^2}{r_j^7} \tag{38}$$

In the preceding equations, $r_j$ and $r_{jZ}$ are, respectively, the geocentric distance and the distance from the satellite $S_j$ to the Earth's equatorial plane, which are expressed in Eqs. (30) and (32). $k_{J2}$ is a constant and is defined in Eq. (8). Furthermore, the orbital rate $\omega_z$, the steering rate $\omega_x$, the orbital acceleration $\alpha_z$, and the steering acceleration $a_x$ of the reference satellite are given in Eqs. (3) and (21–23), respectively. The variables $(r, v_x, h, \theta, i)$ are the CRSV of the reference satellite at time $t$.

*Proof*: Substituting Eq. (28) into the first two terms of Eq. (26), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial K_j}{\partial \dot{\mathbf{q}}_j}\right) - \frac{\partial K_j}{\partial \mathbf{q}_j} = \begin{bmatrix} \ddot{x}_j - 2\dot{y}_j\omega_z - x_j\omega_z^2 - y_j\alpha_z + z_j\omega_x\omega_z - r(\omega_z)^2 + (\dot{v}_x) \\ \ddot{y}_j + 2\dot{x}_j\omega_z - 2\dot{z}_j\omega_x + x_j\alpha_z - y_j\omega_z^2 - y_j\omega_x^2 - z_j\alpha_x + 2v_x(\omega_z) + r(\alpha_z) \\ \ddot{z}_j + 2\dot{y}_j\omega_x + x_j\omega_x\omega_z + y_j\alpha_x - z_j\omega_x^2 + r(\omega_x\omega_z) \end{bmatrix} \tag{39}$$

Replacing the variables in parentheses in Eq. (39) with the expressions [Eqs. (3), (10), (21), and (22)] and using the notations $(\zeta, \eta^2)$ in Eqs. (35) and (37), Eq. (39) is converted to

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial K_j}{\partial \dot{\mathbf{q}}_j}\right) - \frac{\partial K_j}{\partial \mathbf{q}_j} = \begin{bmatrix} \ddot{x}_j - 2\dot{y}_j\omega_z - x_j\omega_z^2 - y_j\alpha_z + z_j\omega_x\omega_z - r\eta^2 - \zeta s_i s_\theta \\ \ddot{y}_j + 2\dot{x}_j\omega_z - 2\dot{z}_j\omega_x + x_j\alpha_z - y_j(\omega_z^2 + \omega_x^2) - z_j\alpha_x - \zeta s_i c_\theta \\ \ddot{z}_j + 2\dot{y}_j\omega_x + x_j\omega_x\omega_z + y_j\alpha_x - z_j\omega_x^2 - \zeta c_i \end{bmatrix} \tag{40}$$

On the other hand, substituting Eq. (33) into the third term in Eq. (26) and using notations $(\zeta_j, \eta_j^2)$ in Eqs. (36) and (38), we get

$$\frac{\partial U_j}{\partial \mathbf{q}_j} = [\,\eta_j^2(r + x_j) + \zeta_j s_i s_\theta \quad \eta_j^2 y_j + \zeta_j s_i c_\theta \quad \eta_j^2 z_j + \zeta_j c_i\,]^T \tag{41}$$

Then, substituting Eqs. (40) and (41) into Eq. (26), the dynamics(34) is established. □

We have the following Remarks about the nonlinear relative dynamics in Proposition 2.

*Remark 2*: No approximation is taken in the derivation, and thus the dynamics (34) is the exact under $J_2$ perturbation. □

*Remark 3*: When we apply the relative model [Eq. (34)] to study the guidance and control problems of satellite formation, the dynamics expressed in Eqs. (9–13) is a good candidate to propagate CRSV $(r, v_x, h, \theta, i)$ under $J_2$ perturbation. □

*Remark 4*: Governed by the relative dynamics (34) and the CRSV dynamics (9–13), the satellite relative motion under $J_2$ perturbation is actually described by 11 first-order differential equations. It is independent of the right ascension of the ascending node $\Omega$. This is because both the two-body and the $J_2$ gravities are axisymmetric and independent of the $\Omega$ motion. □

*Remark 5*: Both the acceleration $\zeta_j$ and the angular velocity $\eta_j$ of the satellite $S_j$ have clear physical meanings. We may decompose the gravity on the satellite $S_j$ along the vectors $\mathbf{r}_j$ and $\mathbf{Z}$ to

$$\mathbf{g}_j = -\nabla U_j = -r_j \eta_j^2 \hat{\mathbf{r}}_j - \zeta_j \hat{\mathbf{Z}} \tag{42}$$

Thus, $\zeta_j$ is the gravity component pointing to the equatorial plane due to $J_2$ perturbation, and $(r_j \eta_j^2)$ is the gravity component pointing to the Earth's center, which is caused by both the two-body and the $J_2$ gravities. Suppose there is a virtual circular orbit at the satellite $S_j$. Then, $\eta_j$ is actually the osculating angular velocity that generates the centrifugal force of the virtual circular orbit. □

## IV. Conclusions

A set of differential equations of the satellite relative motion, which takes into account nonlinearity, eccentricity, and $J_2$ perturbation, has been developed based on Lagrangian mechanics. As a byproduct, a set of simple $J_2$ dynamic equations has also been derived to propagate the motion of the reference satellite under $J_2$ perturbation. Because no approximation is needed in the model development, the developed relative model is exact in the arbitrary eccentric orbits under $J_2$ perturbation. The model is independent of the right ascension of the ascending node and is simply presented in terms of very few physical parameters. It has potential applications to the precise control of fuel-efficient satellite formation flying.

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